Theory of Infectious Disease

Homework 1

Aaron A. King, Robert J. Woods

Due: 29 September 2017

Different forms of the SIR equations for a closed population.

In class, we derived the SIR equations for a closed population—one without births, deaths, or migration—in terms of the *numbers* of hosts in each of the S, I, and R compartments. We had

$$\begin{aligned}\frac{\mathrm{d}X}{\mathrm{d}t} &= -\beta\,\frac{X\,Y}{N}\\ \frac{\mathrm{d}Y}{\mathrm{d}t} &= \beta\,\frac{X\,Y}{N} - \gamma\,Y\\ \frac{\mathrm{d}Z}{\mathrm{d}t} &= \gamma\,Y\end{aligned}$$

where X, Y, and Z are, respectively, the numbers in each of the S, I, and R compartments, and $N = X+Y+Z$ is the total population size. Recall that β is called the *transmission rate* and γ, the *recovery rate*.

(a) Formally change variables to recast the equations in terms of $S = X/N$, $I = Y/N$, and $R = Z/N$.

(b) The above equations assume *frequency-dependent transmission*, i.e., $\lambda = \beta\,Y/N$. Write down the corresponding equations under the assumption of density-dependent transmission, and recast the equations, again, in terms of the fractional occupancy of each compartment.

(c) Compare the resulting equations with those you derived in part (a). Discuss.

Functional form of the force of infection

With respect to an infection you work on, describe verbally and/or mathematically the form of the force of infection. Explain your reasoning. In particular, you can make an argument for frequency- or density-dependent transmission, or something else.